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THE EFFECT OF A STRINGER ON THE STRESS DISTRIBUTION AROUND A HOLE

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We investigate the effect of symmetric stringers, which reinforce a plate in the zone of a circular hole, upon the distribution of the stress field around the hole. The problem reduces to a singular integral equation of the first kind which admits an approximate examination.

An extensive literature is devoted to the problem of the transmission of forces to an elastic body through a stringer. A survey of the results obtained till 1968 is given in [1], where one can find the corresponding bibliographic data. The papers [2–7] belong to the recent investigations devoted to the theoretical aspect of the problem.

We mention that, obviously, the authors of [7] were not aware of the papers [5, 6].

1. Formulation of the problem and notation. An elastic body has the form of an infinite plate with a circular hole. Two identical elastic bars of constant cross section, situated on the same line and with ends on the circumference of the hole are attached (welded) to the plate in the radial direction. The hole is assumed to be free of applied forces. To the ends of the bars at the hole there are applied equal and opposite axial forces and the plate is subjected at infinity to uniaxial extension in the direction of the bars. We assume that the elastic medium is deformed under the conditions of generalized plane state of stress and that the reinforcing bars, called stringers from now on, are idealized one-dimensional continua, deprived of flexural rigidity. There arises the problem of the determination of the effect of the stringers on the distribution of the stresses in the plate around the hole.

For the sake of simplicity, the radius of the hole is taken to be equal to unity. We take the surface of the plate in the plane of the variable $z = x + iy$, the center of the hole in the origin and we place the axes of the stringers along the segments $[-a, -1]$ and $[1, a]$ of the real axis (Fig. 1). The algebraic value of the axial load, applied to the end of the left bar, is denoted by p_0 and the tensile force at infinity by P.

For the elements of the elastic fields and for the characteristics of the plate and the stringer we adopt the following notation: σ_x, σ_y, τ_{xy} denote the stress components in the plane field, u, v are the displacement components, $N(x)$ is the normal force in the section of the stringer, ε° is the relative elongation of the axis of the stringer, E, ν are the elastic constants of the material of the plate, E_0 is the modulus of elasticity of the stringer, S_0 is the area of its cross section and h is the thickness of the

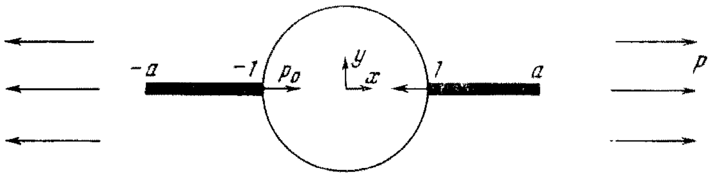

Fig. 1

plate. We introduce also the length, the thickness and the width of the stringer, denoted below by l, h_0 and b, respectively, $l = a - 1$. On this axis of the stringer we will make distinction between the left and the right edge (with respect to the positive direction of the x-axis) and we will assign to the quantities σ_x and so on, relating to them, the signs plus and minus, respectively.

For the solution of the problem we will follow the method indicated in [8] in conformity with the asymmetric case.

2. Boundary conditions. First of all we express the conditions of simultaneous deformations of the plate and the stringers. The condition of the equilibrium of any infinitely small portion of the stringer, fastened to the plate gives two equations

$$h\,(\tau_{xy}^{+} - \tau_{xy}^{-}) + N'(x) = 0, \qquad \sigma_y^{+} - \sigma_y^{-} = 0, \qquad 1 < |x| < a \tag{2.1}$$

The first equality expresses the condition of the vanishing of the projections on the x-axis of all forces applied to the surface of an elementary volume of the stringer contained between two of its cross sections of coordinates x and $x + dx$. The second equality obtained by projecting all forces onto the y-axis, has been derived by taking into account that the stringer does not resist to bending. After integration, the first equality of (2.1) gives

$$h\int_{x_0}^{x} (\tau_{xy}^{+} - \tau_{xy}^{-})\,dt + N(x) - p_0 = 0,\; x_0 = \begin{cases} 1, & 1 < x < a \\ -1, & -a < x < -1 \end{cases} \tag{2.2}$$

Because of the symmetry of the problem we have the following physically obvious relations:

$$\tau_0(-x) = -\tau_0(x), \qquad N(-x) = N(x); \quad 1 < x < a \tag{2.3}$$

$$(\tau_0 = \tau_{xy}^{+} - \tau_{xy}^{-})$$

Therefore in the sequel it is sufficient to restrict ourselves to the equality (2.2) in which $x_0 = 1$.

We make use now of the condition of continuity of the elastic displacements at the axis of the stringer and of the equality of deformations at the same axis. We have

$$u^+ + iv^+ = u^- + iv^- \tag{2.4}$$

$$\left(\frac{\partial u}{\partial x}\right)^+ = \left(\frac{\partial u}{\partial x}\right)^- = \varepsilon^\circ, \qquad 1 < |x| < a \tag{2.5}$$

On the basis of (2.5), making also use of the formula

$$N(x) = E_0 S_0 \varepsilon^\circ(x)$$

well-known from the theory of small deformations of curvilinear beams, the equilibrium condition (2.2) and the second condition (2.1) can be represented in the form of one complex equality in the following manner:

$$h \int_1^x [(\tau_{xy}^+ + i\sigma_y^+) - (\tau_{xy}^- + i\sigma_y^-)]\, dt + E_0 S_0 \left(\frac{\partial u}{\partial x}\right)^+ - p_0 = 0, \; 1 < x < a \tag{2.6}$$

We introduce the Muskhelishvili functions $\varphi(z)$ and $\psi(z)$ and we recall the known formula (see [9], § 33), valid along any profile AB in the field of plane stress (the point A is fixed and B is variable).

$$\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)} = i \int_{AB} (X_n + iY_n)\, ds + \text{const} \tag{2.7}$$

The positive direction on AB is from A to B and the positive normal n to the profile is pointed to the right; $X_n ds$, $Y_n ds$ denote the components of the stress vector acting on the element ds from the side of the positive normal. In the given case, the functions $\varphi(z)$ and $\psi(z)$ are piecewise-holomorphic in the plane outside the hole and have as jump lines the segments $[-a, -1]$ and $[1, a]$.

Formula (2.7) allows us to express the boundary conditions of the problem in terms of the functions φ and ψ. By arguments completely similar to those given for the case of one stringer (see [8], § 31, 33), we can see that the conditions (2.4) and (2.6) along the junction line are equivalent to the following two real equalities:

$$\operatorname{Re}\{\varphi^-(t) - \varphi^+(t)\} = 0 \qquad \text{on } L^+ + L^- \tag{2.8}$$

$$\int_1^x (\tau_{xy}^+ - \tau_{xy}^-)\, dt + K_0 \operatorname{Re} \frac{d}{dx} [\varkappa\varphi(x) - x\overline{\varphi'(x)} - \overline{\psi(x)})] - \frac{p_0}{h} = 0 \tag{2.9}$$

on L^+

$$\varkappa = \frac{3 - \nu}{1 + \nu}, \qquad K_0 = \frac{E_0 S_0}{2\mu h}$$

Here μ is the shear modulus of the material of the plate while L^+ and L^- are the intervals $(1, a)$ and $(-a, -1)$, respectively; by the expression $\varkappa\,\varphi(t) - t\overline{\varphi^1(t)} - \overline{\psi(t)}$, when the point t lies on $L^+ + L^-$, we understand the common limiting values from the left and from the right of the complex displacement $2\mu(u + iv)$ at the corresponding point. The form in which the condition (2.9) is written presupposes the presence of the relation (2.3).

Finally, the condition at the contour of the hole is written in the usual form

$$\varphi(\sigma) + \sigma\overline{\varphi^1(\sigma)} + \overline{\psi(\sigma)} = 0 \quad \text{on } \gamma\ (\sigma = l^{i\theta}) \tag{2.10}$$

where γ is the circumference $|z| = 1$ and σ is a point on it.

The equalities (2.8) – (2.10) exhaust all the conditions of the problem.

3. The construction of the complex potentials. At any point x on the segments L^+ and L^-, where the stringer is joined with the plate, there arises the force $q\ (x)$ directed along the axis of the stringer and equal to

$$q\ (x) = -\overset{+}{\tau}_{xy}\ (x, 0) + \overset{-}{\tau}_{xy}\ (x, 0) \tag{3.1}$$
$$q\ (-x) = -q\ (x)$$

The potentials φ and ψ, corresponding to the force $(q, 0)$ concentrated at the point $t\ (1 < |t| < a)$ and to the tensile force P at infinity, have the form

$$\varphi(z, t) = -p\ (t) \ln \frac{z-t}{z+t} + \frac{\Gamma}{2} z + \varphi_0 (z, t) \tag{3.2}$$
$$\psi(z, t) = p\ (t) \left[\varkappa \ln \frac{z-t}{z+t} + \frac{2zt}{z^2 - t^2} \right] - \Gamma z + \psi_0 (z, t)$$

Here

$$p\ (x) = \frac{q\ (x)}{2\pi (1 + \varkappa)}, \qquad \Gamma = \frac{P}{2}$$

while φ_0 and ψ_0 are functions of z holomorphic everywhere outside the hole including the point at infinity. These latter as well as the functions φ and ψ depend on the real parameter t varying in the intervals L^+ and L^-. The functions φ_0 and ψ_0 will be determined from the boundary conditions (2.10) which by virtue of (3.2) give

$$\varphi_0\ (\sigma, t) + \sigma \overline{\varphi_0'\ (\sigma, t)} + \overline{\psi_0\ (\sigma, t)} = f_0\ (\sigma, t)$$
$$f_0 (\sigma, t) = p\ (t) \left[\ln \frac{\sigma - t}{\sigma + t} - \varkappa \ln \frac{1 - \sigma t}{1 + \sigma t} + \frac{\sigma (\sigma - t)}{1 - \sigma t} - \frac{\sigma (\sigma + t)}{1 + \sigma t} \right] - \Gamma \left(\sigma - \frac{1}{\sigma} \right)$$

The solution of this problem can be represented in the form (see [9], § 82)

$$\varphi_0\ (z, t) = -\frac{1}{2\pi i} \int_\gamma \frac{f_0\ (\sigma, t)\ d\sigma}{\sigma - z}$$
$$\psi_0 (z, t) = -\frac{1}{2\pi i} \int_\gamma \frac{\bar{f}_0\ d\sigma}{\sigma - z} - \frac{\varphi_0'\ (z, t)}{z}$$

These integrals can be easily computed with the Cauchy integral formula and the generalized residue theorem. Performing the necessary computations and substituting the obtained functions φ_0 and ψ_0 into the right-hand side of the formulas (3.2), we find for the functions φ and ψ

$$\varphi\ (z, t) = \Omega_1\ (z, t)\ p\ (t) + g_1\ (z) \tag{3.3}$$
$$\psi\ (z, t) = \Omega_2\ (z, t)\ p\ (t) + g_2\ (z)$$
$$\Omega_1 (z, t) = -\ln \frac{z-t}{z+t} - \varkappa \ln \frac{zt-1}{zt+1} + \frac{2 (1 - t^2) z}{t (1 - z^2)}$$
$$\Omega_2 (z, t) = \varkappa \ln \frac{z-t}{z+t} + \frac{2zt}{z^2 - t^2} + \ln \frac{zt-1}{zt+1} - \frac{2\varkappa t}{z (1 - z^2 t^2)} - \frac{2 (1 - t^2)(1 + z^2 t^2)}{zt (1 - z^2 t^2)^2} - \frac{2}{zt}$$

$$g_1(z) = \frac{\Gamma}{2}\left(z + \frac{2}{z}\right), \qquad g_2(z) = -\Gamma\left(z + \frac{1}{z} - \frac{1}{z^3}\right)$$

On the basis of (3.3), the desired potentials of the problem can be represented in the form

$$\varphi(z) = \frac{1}{2\pi}\int_{L^+} \Omega_1(z, t)\,\tau(t)\,dt + g_1(z) \tag{3.4}$$

$$\psi(z) = \frac{1}{2\pi}\int_{L^+} \Omega_2(z, t)\,\tau(t)\,dt + g_2(z), \quad \tau(x) = 2\pi p(x)$$

From the way the functions $\varphi(z)$ and $\psi(z)$ were constructed, it follows that they satisfy the boundary conditions (2.10) for any real $\tau(x)$. We can also see that condition (2.8) is satisfied exactly. In the case when the stringer is absent, we have: $\tau(x) = 0$ on L^+, $\tau(-x) = -\tau(x)$, and the functions (3.4) give the solution of the well-known Kirsch problem.

4. Reduction to an integral equation. It remains to satisfy the condition given by the equality (2.9). To this end, making use of (3.4), we express the combination $-\varkappa\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)}$ in integral form and we compute the kernel of the integral expression for $z = x$. After elementary computations we obtain for the kernel $\Omega(x, t)$

$$2\pi\Omega(x, t) = \varkappa\,[\ln(x - t) + \overline{\ln(x - t)}] - 2\varkappa\ln(x + t) + (\varkappa^2 + 1)\ln\frac{xt - 1}{xt + 1} +$$
$$2\varkappa\left[\frac{t^2 - 1}{t^2} + \frac{x^2 - 1}{x^2}\right]\frac{xt}{1 - x^2t^2} + \frac{2(1 - t^2)(x^2 - 1)}{xt}\,\frac{1 + x^2t^2}{(1 - x^2t^2)^2} - \frac{2}{xt} \tag{4.1}$$

The right-hand side of this equality, without its first two terms, represents a regular function of the variables x and t for $1 < x,\ t \leqslant a$. Therefore, in computing the derivative

$$d/dx\,[\varkappa\varphi(x) - x\overline{\varphi'(x)} - \overline{\psi(x)}]$$

we can, with the indicated reservation, differentiate under the integral sign. However, for the computation of the derivatives of the terms which contain under the integral the function $\ln(x - t)$ or its conjugate, we have to make use of the formulas

$$\frac{d}{dx}\frac{1}{\pi i}\int_{L^+} \ln(x - t)\,\tau(t)\,dt = -\tau(x) + \frac{1}{\pi i}\int_{L^+}\frac{\tau(t)\,dt}{x - t}$$

$$\frac{d}{dx}\frac{1}{\pi i}\int_{L^+} \overline{\ln(x - t)}\,\tau(t)\,dt = \tau(x) + \frac{1}{\pi i}\int_{L^+}\frac{\tau(t)\,dt}{x - t}$$

which hold for any function $\tau(x)$ continuous on L^+ in the Hölder sense; the integrals on the right-hand sides are considered in the sense of the Cauchy principal value. Performing all the necessary computations and changing under the integral in (2.9) the difference $\tau_{xy}^{+} - \tau_{xy}^{-}$ by its value given by the equality

$$\tau_{xy}^{+} - \tau_{xy}^{-} = -(1 + \varkappa)\,\tau(x) \quad \text{on } L^+$$

we arrive to a singular integral equation relative to $\tau(x)$

$$\frac{1}{2\pi}\int_{L^+}\frac{\tau(t)\,dt}{t - x} + \frac{1}{2\pi}\int_{L^+} k_0(x, t)\,\tau(t)\,dt = f_0(x) \tag{4.2}$$

Here

$$k_0(x,t) = -2\pi\lambda H(x-t) - \frac{1}{2x}K(x,t)$$

$$K(x,t) = \left(\frac{\varkappa^2+1}{x} - \frac{2\varkappa}{x^3}\right)\left[\frac{1}{xt-1} + \frac{1}{xt+1}\right] + \frac{2}{x^2t} - \frac{2\varkappa}{x+t} +$$
$$\left[\varkappa t\left(\frac{t^2-1}{t^2} + \frac{x^2-1}{x^2}\right) - \frac{(t^2-1)(x^2+1)}{x^2t}\right]\left[\frac{1}{(xt-1)^2} + \frac{1}{(xt+1)^2}\right] +$$
$$\frac{2(t^2-1)(x^2-1)}{x}\left[\frac{1}{(xt-1)^3} + \frac{1}{(xt+1)^3}\right]$$

$$f_0(x) = -\frac{\Gamma}{2\varkappa}\left(\frac{\varkappa+1}{2} - \frac{\varkappa+2}{x^2} - \frac{3}{x^4}\right) + Gp_0$$

$$\lambda = \frac{(\varkappa+1)\mu h}{\varkappa S_0 E_0}, \quad G = \frac{\mu}{\varkappa S_0 E_0}, \quad H(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

We introduce the auxiliary complex variable $\zeta = \xi + i\eta$ by the relation

$$z = \alpha\zeta + \beta; \qquad \alpha = l/2, \quad \beta = \alpha + 1$$

where l is the length of the stringer. After such a substitution, the axis of the (right) stringer turns into the segment [−1, 1] of the real axis ξ and the outline $|z| = 1$ of the hole in the physical plane turns into a circumference of radius $2/l$ with center at the point $\zeta = 1 - 2/l$. Equation (4.2) becomes

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{\nu(\eta)\,d\eta}{\eta-\xi} + \frac{1}{2\pi}\int_{-1}^{1}k(\xi,\eta)\,\nu(\eta)\,d\eta = f(\xi) \tag{4.3}$$

$$k(\xi,\eta) = \alpha k_0(x,t), \qquad \nu(\xi) = \tau(x), \qquad f(\xi) = f_0(x)$$

$$x = \alpha\xi + \beta, \qquad t = \alpha\eta + \beta$$

After finding the solution $\tau(x)$ of Eq. (4.2), the formulas (3.4) give the solution of our problem.

5. The computation scheme. We assume that the singular equation (4.3) has a solution $\nu(\xi)$ satisfying the Hölder condition at any closed part of the segment [−1, 1] not containing the endpoints $\xi = \pm 1$ and having at the endpoints of the segment singularities of order less than unity. The scheme of the approximate solution of an equation of the form (4.3) is given explicitly in ([8], § 13, 33).

We seek the solution in the form

$$\nu(\xi) = (1+\xi)^{1/2}(1-\xi)^{-1/2}\nu^\circ(\xi) \tag{5.1}$$

where $\nu^\circ(\xi)$ is a continuous function on the segment without the left endpoint, approximated by a Lagrange polynomial constructed over the Chebyshev nodes (n is a natural number)

$$\xi_m = \cos\vartheta_m, \quad \vartheta_m = \frac{2m-1}{2n}\pi, \quad m = 1, 2, \ldots, n \tag{5.2}$$

As it is known, the approximating polynomial has the form

$$\nu^\circ(\xi) \cong \mathrm{Ln}[\nu^\circ;\xi] = \frac{1}{n}\sum_{k=1}^{n}(-1)^{k+1}\nu^\circ(\xi_k)\frac{\cos n\vartheta\sin\vartheta_k}{\cos\vartheta - \cos\vartheta_k} \quad (\xi = \cos\vartheta) \tag{5.3}$$

The representations (5.1) and (5.3) lead to a quadrature formula for the singular integral (see [8], formula (13.13))

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{\nu(\eta)\,d\eta}{\eta-\xi}=\frac{1+\cos\vartheta}{n\sin\vartheta}\sum_{k=1}^{n}\nu^\circ(\xi_k)\sum_{m=0}^{n-1}\cos m\vartheta_k\sin m\vartheta+\frac{1}{2n}\sum_{k=1}^{n}\nu^\circ(\xi_k) \quad (5.4)$$

For the computation of the integrals contained in (4.3) in the usual sense, we will make use of the Gauss-type formula (for example, [10], p. 614 Russian edition)

$$\frac{1}{2\pi}\int_{-1}^{1}\frac{F(\xi)\,d\xi}{\sqrt{1-\xi^2}}=\frac{1}{2n}\sum_{k=1}^{n}F(\cos\vartheta_k) \quad (5.5)$$

The quadrature formulas (5.4) and (5.5) allow us to replace Eq. (4.3) by a system of linear equations relative to the approximative values of $\nu^\circ(\xi)$ at the interpolation nodes

$$\sum_{k=1}^{n}\alpha_{mk}\nu_k^\circ=f_m, \qquad m=1,2,\ldots,n \quad (5.6)$$

Here

$$\alpha_{mk}=\frac{1}{2n}\left[1+\operatorname{ctg}\frac{\vartheta_m}{2}\operatorname{ctg}\frac{\vartheta_m\mp\vartheta_k}{2}+(1+\cos\vartheta_k)\,k(\cos\vartheta_m,\cos\vartheta_k)\right]$$
$$\nu_m^\circ=\nu^\circ(\cos\vartheta_m), \qquad f_m=f(\cos\vartheta_m) \quad (5.7)$$

The upper sign in (5.7) is taken in the case when the number $|m-k|$ is odd and the lower sign when it is even (zero is considered an even number). After solving the system (5.6), the approximate solution of Eq. (4.3) is determined by the formulas (5.1) and (5.3).

We turn to the computation of the physical quantities. Apparently, the greatest interest represents the determination of the effect of the stringer on the magnitude of the tensile stress σ_ϑ along the contour of the hole. We recall the known formula

$$\sigma_\vartheta+\sigma_r=4\operatorname{Re}\varphi'(z) \quad (5.8)$$

which gives the sum of the normal stresses at any point of the domain occupied by the elastic medium. In the case under consideration, when applied forces are not acting along the boundary of the hole, the limiting values of the right-hand side on γ, which exist everywhere except at points $z=\pm 1$ will give the value of σ_ϑ at the corresponding point of the circumference. From the first formula of (3.4) and the expression of $\Omega_1(z,t)$ from (3.3) we find by differentiation

$$\varphi'(z)=\frac{1}{\pi}\int_{L^+}M(z,t)\,\tau(t)\,dt+\frac{\Gamma}{2}\left(1-\frac{2}{z^2}\right)$$
$$M(z,t)=\frac{z}{t^2-z^2}+\frac{\varkappa t}{1-z^2t^2}+\frac{(1-t^2)(1+z^2t^2)}{t(1-z^2t^2)^2}$$

Obviously, the function $\varphi'(z)$ is holomorphic everywhere outside the hole, except for the points of the line L^++L^-. The contour stresses σ_ϑ attain a maximum in absolute value at the points $z=\pm i$. We can see that the function $\varphi'(z)$ has at these points the same real part and their common value is given by the formula

$$\operatorname{Re}\varphi'(z)=\frac{1}{\pi}\int_{L^+}m(t)\,\tau(t)\,dt+\frac{3}{2}\Gamma \qquad \text{for } z=\pm i$$

$$m(t) = \frac{(\varkappa+1)t}{t^2+1} + \frac{(t^2-1)^2}{t(t^2+1)^2}$$

The integral, by virtue of the continuity of $m(t)$ is computed with the formula (5.5)

$$\frac{1}{\pi}\int_{L^+} m(t)\,\tau(t)\,dt = \frac{\alpha}{\pi}\int_{-1}^{1} m_0(\xi)\,\nu(\xi)\,d\xi = \frac{\alpha}{n}\sum_{k=1}^{n}(1+\cos\vartheta_k)\,m_0(\cos\vartheta_k)\,\nu_k^\circ$$

$$m_0(\xi) = m(t) \qquad (t = \alpha\xi + \beta)$$

According to the indicated scheme, in the case of simple extension at infinity: $p_0 = 0$, $P = 1$, we have computed the maximum contour stresses σ_θ as function of the ratio of the reduced rigidities $m = E_0h_0 / Eh$. For the other parameters of the problem we have taken (ν is the Poisson's ratio): $\nu = 1/_3$, $b = 0.2$, $l = 1$.

Below we give the numerical values of $\sigma_0 = \max\sigma_\theta$, obtained by solving (5.6) with $n = 40$, $m = 1,2,\ldots,9$

2.2200, 2.2649, 2.3007, 2.3292, 2.3522, 2.3710, 2.3866, 2.3998, 2.4109

Clearly, the maximum stress σ_θ decreases with the decrease of the relative rigidity m The computations show that for a further decrease of m, as some value m_0 less than unity is reached, an opposite dependence occurs. For $m \to 0$, the maximum value σ_0 approaches the number three.

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